**B.Sc. Part—II Semester—IV Examination**

**MATHEMATICS**

**(Classical Mechanics)**

**Paper—VIII**

Time : Three Hours] [Maximum Marks : 60

**Note :—**(1) Question No. **1** is compulsory and attempt it once only.

(2) Solve **ONE** question from each Unit.

1. Choose the correct alternative :

(1) A bead sliding along the wire. The constraint is :

(a) Holonomic (b) Non-holonomic

(c) Both holonomic and non-holonomic (d) None of these

(2) The system of particles will be in equilibrium if the virtual work done by the applied forces is :

(a) Infinity (b) Zero

(c) Non-zero (d) None of these

(3) A point on the orbit of a planet that is nearest to the Sun is called _____ of planet.

(a) Aphelion (b) Apse

(c) Perihelion (d) Focus

(4) Let $\dot{f} = f(q, \dot{q}, t)$. Then $\Delta f = \delta f$ when :

(a) f does not contain q (b) f does not contain $\dot{q}$

(c) f does not contain t explicitly (d) f is constant in time t

(5) In the frame rotation, the co-ordinate frame :

(a) Held fixed (b) Rotates

(c) Orthogonal (d) None of these

(6) The general displacement of a rigid body with one point fixed is :

(a) Rotation about some axis (b) Rotation about some points

(c) Rotation about focus (d) None of these

(7) In $\delta$-variation Hamiltonian H :

(a) Constant (b) Varies

(c) Zero (d) None of these

(8) The second order partial differential equation $F_z - \frac{\partial}{\partial x} F_p - \frac{\partial}{\partial y} F_q = 0$ is known as :

(a) Euler Poisson equation (b) Euler–Ostrogradsky equation

(c) Geodesics (d) None of these

(9) Area of triangle with side $\overline{a}$ and $\overline{b}$ is :

(a) $\overline{a} \circ \overline{b} = 2$ (b) $\frac{1}{2}|\overline{a} \times \overline{b}|$

(c) $\frac{1}{2}\overline{a} \circ \overline{b}$ (d) None of these

(10) The constraints on a bead on uniformly rotating wire is a free space is :

(a) Rheonomous (b) Scleronomous

(c) Rheonomous and Scleronomous (d) None of these 1×10=10

**UNIT—I**

2. (a) Obtain the equation of motion of a simple pendulum by using D'Alembert's principle. 5

(b) Obtain the Lagrange's equation of motion for the double pendulum of length $\ell_1$ and $\ell_2$ with corresponding masses $m_1$ and $m_2$. 5

3. (p) Find the Lagrangian for the system consisting of a simple pendulum of mass $m_2$ with mass $m_1$ at the point of support which can move on horizontal line lying in the plane in which $m_2$ moves. 5

(q) For one-dimensional system consisting of a particle the generalized force Q can be obtained from the potential V in the usual way i.e. $Q = \frac{-\partial V}{\partial q}$. Show that for a velocity independent potential Lagrange's equation can be written in the form $\frac{d}{dt}\frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} = Q$. 5

## UNIT—II

4. (a) Prove that for a central force field F, the path of a particle of mass m is given by

$$\frac{d^2u}{dQ^2} + u = -\frac{m}{h^2u^2} F\left(\frac{1}{u}\right), u = \frac{1}{r} \ . \qquad 5$$

(b) Show that for a particle moving under a central force such that $V = k r^{n+1}$, the virial theorem reduces to $\overline{2T} = (n+1)\overline{V}$. Also prove that for an inverse square law, $\overline{2T} = -\overline{V}$. 5



5. (p) Show that the equation of the orbit can be put in the form $r = \frac{a(1-\in^2)}{1+\in \cos \alpha\theta}$ of a particle moving in a central force field $F = -\frac{k}{r^2} + \frac{c}{r^3}$. Furthermore show that it is an ellipse for $\alpha = 1$. 5



(q) Show that if a particle describes a circular orbit under the influence of an attractive central force directed towards point on a circle, then the force varies as the inverse fifth power of the distance. 5

## UNIT—III



6. (a) Define distance between curves. Find the distance between curves $y(x) = x \cdot e^{-x}$, $y_1(x) = 0$ on [0, 2]. 1+4

(b) Show that the functional $I[y(x)] = \int_0^1 x^3 \sqrt{1 + y^2(x)}\, dx$ define on the set of function $y(x) \in C[0, 1]$ is continuous on the function $y_0(x) = x^2$ in the sense of Zeroth-order proximity. 5

7. (p) If x does not occur explicitly in f, then prove that $f_{y'}\, y' - f = \text{constant}$. 5

(q) Find the extremals of :

$$I[y(x)] = \int_a^b [y^2 + y'^2 + 2ye^x]\, dx . \qquad 5$$

## UNIT—IV

8. (a) Define Hamiltonian H. Prove that cyclic co-ordinate will be absent in Hamiltonian. 1+4



(b) Deduce the Hamiltonian's equation of motion of a particle of mass m in spherical polar co-ordinates $(r, \theta, \phi)$. 5

9. (p) Obtain the Hamiltonian and then deduce the equations of motion for a simple pendulum. Show that the Hamiltonian of the system is the total energy and also the constant of motion. 5

(q) Define Routhian. Prove that a cyclic co-ordinate will not occur in the Routhian R. 5

## UNIT—V

10. (a) Define infinitesimal rotation. Prove that infinitesimal rotation matrix $\in$ is antisymmetric. 1+4

(b) Prove that $3 \times 3$ matrix A is a rotation matrix, then A is orthogonal and $|A| = 1$. 5

11. (p) Prove that the change in the components of a vector $\bar{r}$ under the infinitesimal transformation of the co-ordinate system can be expressed as $d\bar{r} = \bar{r} \times d\bar{u}$, where $d\bar{u} = (du_1, du_2, du_3)$ is vector specifying an infinitesimal rotation. 5

(q) If A is any $2 \times 2$ orthogonal matrix with determinant $|A| = 1$ then prove that A is a rotation matrix. 5